Hydraulic Design of Gas or Vapor Piping Systems

Circular pipe is the usual means moving gas or vapor in process plants, utility systems, or pipelines. We will, therefore, emphasize here the design of piping system for gas or vapor of constant viscosity flowing through circular pipes or tubes.

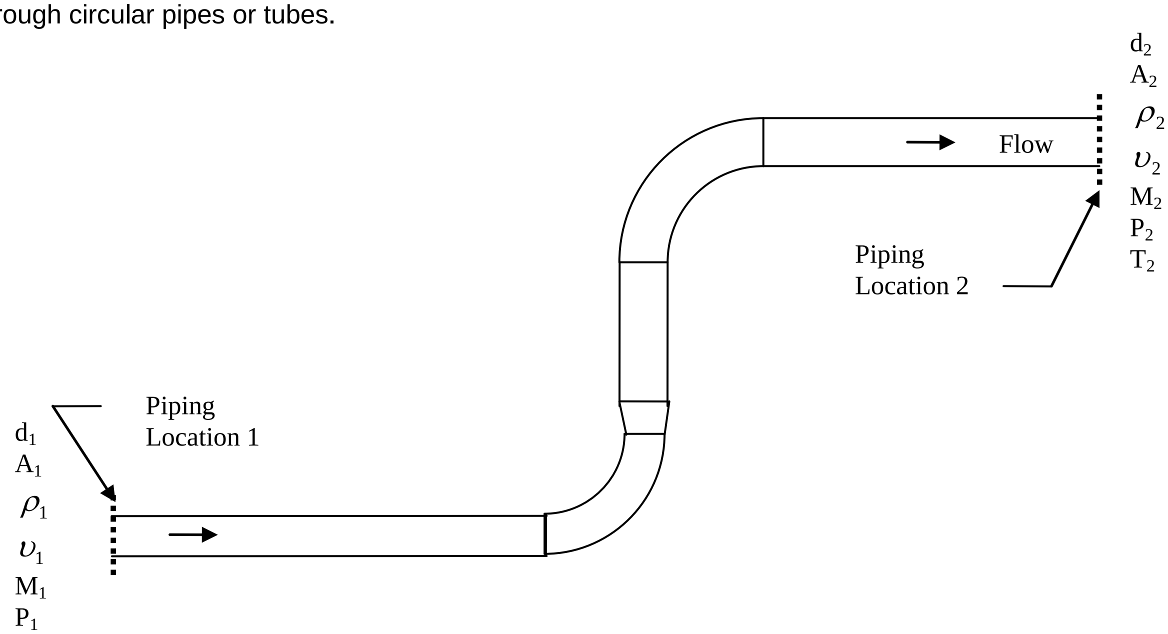


Fig. 1 A Simple Gas Piping System

Density of the Flowing Gas or Vapor ρ

As opposed to a liquid which is considered incompressible in a pipe flow hydraulic analysis, a gas or vapor contracts or expands more pronouncedly upon change in pressure. Thus, while the density is fairly constant over a practical range of pressures in a liquid pipe flow system, the density of a gas or vapor, and consequently its flowing velocity, changes when it flows along the pipe. The term '**compressible**' is used to describe the gas or vapor fluid flow. It is therefore prudent to consider the effect change in gas density as an improvement of accuracy over the incompressible hydraulic equation for gas or vapor.

Density of a gas or vapor is directly a function of temperature and pressure at any given point in the pipe. To estimate the gas density at the flowing T T and P, an "equation of state", which governs the pressure-density-temperature relationship is therefore needed.

Ideal Gas Behavior

For example, an equation of state for an ideal gas, which is followed closely by many gases of low molecular weight and/or at lower pressures, can be used to calculate the flowing density:

$$\rho_{lb/CF} = \frac{(\mathrm{M_W})(\mathrm{P}_{psia})}{(\mathrm{R}_{10.73})(\mathrm{T}_{\deg F} + 459.7)} \quad \text{for an ideal gas} \tag{1}$$

where

$\mathrm{M_W}$ = molecular weight for the flowing gas, lb/lbmol

R =universal gas constant = $10.73\frac{(\text{psia})(\text{ft}^3)}{(\text{lbmol})(^\circ\text{R})}$

Real Gas Behavior

At higher pressures, the ideal gas density equation Eq (1) normally is not adequate. To account for this non-ideality behavior, a third parameter, compressibility factor Z, is commonly added to account for the departure from the ideal gas behavior:

$$\rho_{lb/CF} = \frac{(\text{M}_\text{W})(\text{P}_{psia})}{(\text{R}_{10.73})(z)(\text{T}_{\text{deg}F} + 459.7)} \quad \text{for a real gas} \tag{2}$$

The compressibility factor Z typically approaches 1.0 at lower pressure for real gases (i.e., a real gas behaves more like an ideal gas at lower pressures).

The gas specific gravity S_g frequently appears in the hydraulic equations and is defined below:

$$\text{S}_\text{g} = \frac{\text{M}_\text{W} \text{ of Gas}}{\text{M}_\text{W} \text{ of Air}} = \frac{\text{M}_\text{W} \text{ of Gas}}{28.9} \tag{3}$$

Mass Balance

Even through the flowing density of the gas would change along the pipe, the mass flow rate in a piping system will not change unless addition or diversion of flow occurs between two flow locations (Location 1 and Location 2 in the above sketch). This can be demonstrated by Eq (3):

$$\text{W}_{lb/hr} = (\rho_{lb/CF})_1 (\text{A}_{ft^2})_1 (\upsilon_{ft/\sec})_1 = (\rho_{lb/CF})_2 (\text{A}_{ft^2})_2 (\upsilon_{ft/\sec})_2 \tag{4}$$

where W =flow rate in pounds per hour, lb/hr

ρ = flowing density, pounds per cubic foot, lb/CF;

A_{ft^2} = pipe cross-sectional area in square feet, ft^2

$$\text{A}_{ft^2} = \frac{\pi}{4}\left(\frac{\text{d}_{inch}}{12}\right)^2 \text{ for a circular pipe.} \tag{5}$$

d = inside diameter of the pipe, inches.

υ = flowing velocity in pipe in feet per second, fps.

$$\upsilon_{fps} = \frac{W_{lb/hr}}{(\rho_{lb/CF})(3600_{sec/hr})(A_{ft^2})}$$
$$= (0.0864)\frac{(q_{scfm})(z)(T_{\deg F} + 459.7)}{(P_{psia})(d_{inch})^2}$$
$$= (0.00144)\frac{(q_{scfh})(z)(T_{\deg F} + 459.7)}{(P_{psia})(d_{inch})^2} \quad (6)$$
$$= 60\frac{(Q_{MMscfd})(z)(T_{\deg F} + 469.7)}{(P_{psia})(d_{inch})^2}$$

q_{scfm} = volumetric flow rate in standard cubic feet per minute (@14.7 psia & 60^{o}F)

q_{scfh} = volumetric flow rate in standard cubic feet per hour (@14.7 psia & 60^{o}F)

Q_{MMscfd} = volumetric flow rate in million standard cubic feet per day (@14.7 psia & 60^{o}F)

Thus, even through the cross-sectional areas of the pipe at Location 2 (downstream) and Location 1 (upstream) are the same, the velocity at Location 2 will be changed from that at Location 1 simply because of the change in pressures due to frictional pressure drop between those two locations.

Now we want to introduce two useful parameters associated with the compressible fluid that will appear later in the pressure drop calculation methods, the sonic velocity υ_S and the Mach number M.

Sonic (or Critical) Velocity in Pipe

For compressible fluids, the flowing fluid's pressure drop will cause the pipe downstream velocity to increase. The maximum possible velocity that flow can attain in a pipe is the velocity of sound in that fluid:

$$\upsilon_s = \sqrt{\frac{(g_c)(144)(R_{10.73})(k)(z)(T_{^\circ R})}{(M_W)}} = (223)\sqrt{\frac{(k)(z)(T_{^\circ R})}{(M_W)}} \quad (7)$$

where

$T_{^\circ R}$ = temperature of fluid at which the sonic velocity occurs.

A sufficiently high upstream pressure or a sufficiently low downstream pressure can cause the sonic velocity to occur in the outlet end of a pipe or in the downstream of an expanded area. Note also that the sonic velocity will be developed in a pipe line even before its exit if the line is sufficiently long under the above conditions. In this situation, the pressure somewhere in the pipe line at which the sonic velocity occurs can be higher than that at the exit of the pipe line; and the remaining "surplus" pressure drop (from sonic pressure to the exit pressure) will not further increase the flow rate but, in stead, will be consumed in the turbulence of eddies.

Mach number

Mach number is defined as ratio of velocity of fluid to velocity of sound in that fluid:

$$\begin{aligned}\text{Mach Number} = M = \frac{\upsilon}{\upsilon_s} &= \frac{(R_{10.73})}{(3600)(223)} \frac{W_{lb/hr}}{(P_{psia})(A_{ft^2})} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} \\ &= 0.00001336 \frac{W_{lb/hr}}{(P_{psia})(A_{ft^2})} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} \\ &= 1.702\text{x}10^{-5} \frac{W_{lb/hr}}{(P_{psia})(D_{ft})^2} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} \\ &= \frac{1}{408} \frac{W_{lb/hr}}{(P_{psia})(d_{inch})^2} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}}\end{aligned} \tag{8}$$

Critical velocity (or sonic velocity) is defined as the velocity at the point where Mach number reaches unity.

Maximum Capacity of Pipe

The maximum mass flow rate that can flow through a pipe line will be limited by the sonic pressure if it is developed before or at the exit of the pipe where the fluid's flowing velocity at the sonic pressure reaches the sonic velocity of the flowing fluid. To express this maximum mass flow rate, we can use the above Mach Number equation Eq (8) and set P_{psia} to $P_{cf,psia}$ and the Mach number to 1.0. Rearrange it to obtain the maximum flow rate $W_{cf,}$ in a pipe or pipeline as a function of the sonic pressure $P_{cf,psia}$:

$$\begin{aligned}\frac{W_{cf,\ lb/hr}}{(A_{ft^2})} &= \frac{(223)(3600)}{(R_{10.73})} P_{cf,psia} \sqrt{\frac{(k)(M_W)}{(z)(T_{^\circ R})}} \\ \frac{W_{cf,\ lb/hr}}{(A_{ft^2})} &= \frac{1}{0.0001336} P_{cf,psia} \sqrt{\frac{(k)(M_W)}{(z)(T_{^\circ R})}} \\ \frac{W_{cf,\ lb/hr}}{(D_{ft})^2} &= (1.702\text{x}10^{-5}) P_{cf,psia} \sqrt{\frac{(k)(M_W)}{(z)(T_{^\circ R})}} \\ \frac{W_{cf,\ lb/hr}}{(d_{inch})^2} &= (408) P_{cf,psia} \sqrt{\frac{(k)(M_W)}{(z)(T_{^\circ R})}}\end{aligned} \tag{9}$$

For an ideal-gas isothermal flow segment of constant inside diameter, the material balance equation Eq (4) and Equation of State Eq (1) lead to the following relationship for the Mach number at entrance and exit:

$$\frac{\text{Mach number @ Inlet}}{\text{Mach number @ Outlet}} = \frac{\rho_{lb/CF}\text{ @ Outlet}}{\rho_{lb/CF}\text{ @ Inlet}} = \frac{P_{psia}\text{ @ Outlet}}{P_{psia}\text{ @ Inlet}} \tag{10}$$

Example 1

How to detect if sonic velocity will occur in a pipe before its exit?

At any given flow rate $W_{lb/hr}$ in a pipe of ID = d_{inch}, the critical flow will occur when the pressure anywhere in pipe deceases to $P_{cf\ psia}$ or the Mach number increases to 1.0 before the pipe exit. Thus we can use one of the following, by setting $M_2 = 1$ in Eq (8), to detect whether $W_{lb/hr}$ will cause sonic flow:

$$\frac{(R_{10.73})}{(3600)(223)} \frac{W_{lb/hr}}{(A_{ft^2})} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} > P_{@Exit}$$

$$0.00001336 \frac{W_{lb/hr}}{(A_{ft^2})} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} > P_{@Exit} \qquad (11)$$

$$1.702x10^{-5} \frac{W_{lb/hr}}{(D_{ft})^2} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} > P_{@Exit}$$

$$\frac{1}{408} \frac{W_{lb/hr}}{(d_{in})^2} \sqrt{\frac{(z)(T_{^\circ R})}{(k)(M_W)}} > P_{@Exit}$$

As will be show in the later sections of this course, at the sonic condition the frictional pressure drop across the pipe can be calculated by the rational formula:

$$\Delta P_{f,\ psi} = P_{upstream,\ psia} - P_{cf,\ psia} = \left[\frac{f L_{ft}}{D_{ft}} + \sum_i K_i + \ln\left(\frac{P_{1,\ psia}}{P_{2,\ psia}}\right)^2 \right] \frac{(P_{1,\ psia})^2 k M_1^2}{2P_{avg,\ psia}}$$

where $P_{avg,\ psia} = \dfrac{P_{1,psia} + P_{2,psia}}{2}$;

or by the modified Darcy's equation

$$\Delta P_f = P_{upstream} - P_{cf} = \left[\sum_i K_i + f \frac{L}{D} \right] \frac{1}{Y^2} \frac{(\rho)(\upsilon^2)}{(2)(g_c)(144)}$$

where Y = net expansion factor.
All parameters are evaluated at the upstream conditions.

Any pressure lower than P_{cf} (i.e., from P_{cf} to $P_{@Exit}$) will be lost in shock waves and turbulence instead of being converted into useful kinetic energy. In other words, if the pipe outlet is exposed to another environment which has a pressure that is lower than the calculated P_{cf}, the pressure at the outlet end of pipe will be P_{cf}; and the remaining pressure drop (from P_{cf} to the pressure in the downstream environment) will be wasted.

Overall Pressure Balance

The overall pressure balance equation for each flow segment of constant inside diameter flowing an incompressible fluid is

$$\Delta P + \Delta P_s + \Delta P_p = \Delta P_f + \Delta P_{acc} \quad (12)$$

or

$$P_{in} - P_{out} + \left[\frac{(\rho)(h)}{144}\right]_{in} - \left[\frac{(\rho)(h)}{144}\right]_{out} + \Delta P_p = \Delta P_f + \Delta P_{acc}$$

where

P_{in} = pressure at the inlet of the segment, psig

P_{out} = pressure at the outlet of the segment, psig

ΔP = system total pressure drop = $P_{in} - P_{out}$, psi

ΔP_s = segment static pressure change due to change in elevation , psi

$$\Delta P_{s\ psi} = \left[\frac{(\rho_{lb/CF})(h_{ft})}{144}\right]_{in} - \left[\frac{(\rho_{lb/CF})(h_{ft})}{144}\right]_{out}$$

ΔP_p = pressure increase due to compressor, psi

ΔP_f = segment frictional pressure drop, psi

ΔP_{acc} = segment pressure drop due to fluid acceleration, psi

ρ = flowing density, pounds per cubic foot, lb/CF;

h = elevation, ft

Acceleration pressure drop, ΔP_{acc}, can be appreciable for a compressible flow when the pressure drop and the increase in flowing velocity is substantial. As will be shown later in this course, the calculation of frictional pressure drop of compressible flow will include the effect of acceleration. Furthermore, we will not address the compressor flow in this study guide. Consequently, $\Delta P_p = 0$.

The overall pressure balance equation for a gas flowing through a series of pipes becomes:

$$P_{in} - P_{out} + \sum\left[\frac{(\rho_{in} h_{in} - \rho_{out} h_{out})}{144}\right]_{upflow} + \sum\left[\frac{(\rho_{in} h_{in} - \rho_{out} h_{out})}{144}\right]_{downflow} = \sum \Delta P_f \quad (13)$$

While Eq (13) is useful for an isothermal flow system, it is applicable to a flowing system under hydraulic analysis whose flowing temperature undergoes changes such as flowing through an heat exchanger. We can calculate the ΔP_f for each isothermal flow segment and then also sum them up to obtain the cumulative frictional pressure drop $\sum \Delta P_f$ for the entire system.

Hydraulic Terminology in Pipe Flow

Some frequently used hydraulic terminologies relating to the word "**head**" (for pressure) are introduced below:

- **Pressure, Pressure Head**

  Pressure is the force exerted by fluid that acts to pipe or container and can be expressed as force per square unit surface area. This pressure typically can be measured directly by a pressure gage in a unit such as pounds per square inch, psig.

  Parallel to the principle of a mercury barometer, the same pressure that is exerted by the fluid also can cause a measuring fluid to rise in a column to a height of H_{ft} until its weight balances out the pressure in the pipe or container. In other words, a column of liquid will exert a local **Pressure** that is proportional to the height of that liquid:

$$\mathrm{P}_{psia} = \frac{(\rho_{lb/CF})(\mathrm{H}_{ft})}{144} \qquad (14)$$

  Thus, the pressure in the pipe or container can be expressed by the height of the column of a measuring gas, such as mercury, water or the same fluid itself, and is called **Pressure Head**:

$$\mathrm{H}_{ft} = \frac{(144)(\mathrm{P}_{psia})}{\rho_{lb/CF}} \qquad (15)$$

  For example, normal atmospheric pressure at 0 psig or 14.6959 psia and 60^{o}F is equivalent to a head of mercury of 29.92 (=14.69X144/848.7X12) inches or a head of water of 33.93 (=14.6959X144/62.3688) feet. Therefore, the pressure will increase by 2.036 psi per inch of mercury column rise or 0.433 psi per foot of water column height:

  14.6959 psi = 29.92" Hg = 760 mmHg
  1 psi = 2.036"Hg
  0.433 psi = 1' WC (60^{o}F, density = 62.3688 lb/CF)
  1 psi = 2.31' WC (60^{o}F, density = 62.3688 lb/CF)

- **Static Pressure, Static Head**

  Static head is a measurement of pressure due to the weight of column of fluid at a given height. Thus, a fluid at a level of H feet above a reference point (a difference in elevation) is said to have a static head of $\mathrm{H}_{s,\ ft}$. This **Static Head** of H feet also can be expressed in terms of a local pressure at the base of the column if the density of the fluid is known, for example,

$$\mathrm{H}_{s,\ psi} = \frac{(\rho_{lb/CF})(\mathrm{H}_{s,\ ft})}{144} \text{ in pounds per square inch of area.} \qquad (16)$$

  Static head $\mathrm{H}_{s,\ psi}$ also can be called **Static Pressure** simply because it measures the pressure head and is expressed in a pressure unit.

- **Velocity Head**

We will use the following simplified concept for the purposes of introducing the term velocity head. When an object falls, it will lose its potential energy. It will, in turn, gain the kinetic energy at the expense of potential energy. Thus, when a fluid falls from a stationary state for H feet, its velocity will increase from zero to υ ft/sec at a kinetic energy of $\rho v^2 / 2g_c$ in ft-lb per cubic feet. We say that the static head $H_{s,\ ft}$ produces a **Velocity Head**

$$H_{v,\ ft} = \frac{(\upsilon_{ft/\sec})^2}{(2)(g_c)} \quad \text{in feet,} \tag{17}$$

where

g_c = gravitational constant, 32.18 ft/sec^2.

Thus, Eq. (17) defines the concept of velocity head.

Eq. (17) can be converted to a pressure unit such as

$$H_{v,\ psi} = \frac{(\rho_{lb/CF})(\upsilon_{ft/\sec})^2}{(2)(g_c)(144)} \quad \text{in unit of pounds per square inch, psi} \tag{18}$$

Again, velocity head $H_{v,\ psi}$ also can be called **Velocity Pressure** because of it unit.

Note that in a pipe segment of constant inside diameter flowing liquid or water, the velocity, and hence the velocity head, do not change throughout the segment.

One of the utilities of velocity head in pipe flow hydraulics is to help express the frictional pressure drop of equipment, such as a flow nozzle, heat exchanger, etc. in terms of number of velocity heads (see Eq. 28, 33).

- **<u>Static Head-Loss</u>**
  When a pipe segment runs inclined or vertical, there will be a difference in elevation between the start and the end of that pipe segment. This difference in elevation Δh (=inlet elevation - outlet elevation) will result in a pressure difference due to weight of fluid regardless fluid is flowing or not. **Static Head-Change** can be expressed either in pressure unit or in head unit:

$$\Delta H_{s,\ psi} = \Delta P_{s,\ psi} = \frac{(\rho_{lb/CF})(\Delta h_{ft})}{144} \tag{19}$$

$$\Delta H_{s,\ ft} = h_{in,ft} - h_{out,ft} = \Delta h_{ft} = \frac{(144)(\Delta P_{s,\ psi})}{\rho_{lb/CF}}. \tag{20}$$

In down flow where ΔP_s or ΔH_s is positive, we say that there is a **Static Head Gain**.

In up flow where ΔP_s or ΔH_s is negative, we say that there is a **Static Head Loss**.

Since we take static head loss as a positive quantity, Static Head Loss = -ΔH_s =-ΔP_s .

When an overall piping circuit involves several pipe segments which has its own elevation change, these individual elevation changes can be added together to arrive at the overall elevation change. In other words, the static head change is additive and the net static head change for the overall system of constant density depends on only the elevation at the start and the end of that piping circuit.

- **Dynamic Head-Loss**

  The term **Dynamic Head-Loss** or **Dynamic Pressure Drop** refers to the frictional pressure drop associated with piping (pipe and flow elements) plus equipment:

$$\Delta H_{f,\ psi} = \Delta P_{f,\ psi} \tag{21}$$

$$\Delta H_{f,\ ft} = \frac{(144)(\Delta P_{f,\ psi})}{\rho_{lb/CF}} \tag{22}$$

The formula for $\Delta P_{f,\ psi}$ will be presented later.

- **Total Head-Loss**

$$\text{Total Head Loss} = \text{Dynamic Head Loss} + \text{Static Head Loss} \tag{23}$$

Total head loss is also called **System Pressure Drop**.

Finally, the overall pressure balance equation, Eq. 12, for a compressible fluid, such as a gas, flowing through a series of pipes can be written in terms of heads:

$$\left(\frac{(P_{in,\,psia})(144)}{\rho_{in\ \ lb/CF}} - \frac{(P_{out,\,psia})(144)}{\rho_{out\ \ lb/CF}}\right)_{ft} + \sum\left[h_{in,\,ft} - h_{out,\,ft}\right]_{upflow} + \sum\left[h_{in,\,ft} - h_{out,\,ft}\right]_{downflow} = \sum\left(\frac{(\Delta P_{f,\,psi})(144)}{\rho_{avg\ \ lb/CF}}\right)_{ft}$$

Total Head Loss - Static Head Loss Head Loss Dynamics

(24)

Reynolds number Re

This is the parameter used to establish the flow regimes in a pipe. The value of friction factor, which is used to correlate the dynamics head loss, depends on the Reynolds number

Reynolds number	Flow Regime
< 2,000	Laminar
2,000 to 4,000	Transition
> 4,000	Turbulent

$$\mathrm{Re} = \frac{(\rho_{lb/CF})(\mathrm{D}_{ft})(\upsilon_{ft/\sec})}{(\mu e_{\mathrm{lb/ft/sec}})} = 6.31\frac{\mathrm{W}_{lb/hr}}{(\mathrm{d}_{\mathrm{inch}})(\mu_{cP})}$$

$$= \frac{(\mathrm{q}_{scfm})(\mathrm{M_W})}{(\mathrm{d}_{\mathrm{inch}})(\mu_{cP})} = (0.0167)\frac{(\mathrm{q}_{scfh})(\mathrm{M_W})}{(\mathrm{d}_{\mathrm{inch}})(\mu_{cP})} = (693.4)\frac{(\mathrm{Q}_{MMSCFD})(\mathrm{M_W})}{(\mathrm{d}_{\mathrm{inch}})(\mu_{cP})} \qquad (25)$$

Re = Reynolds number
μ_e = absolute viscosity, lb/ft/sec
μ = viscosity, centipoise, cP
D = inside diameter of the pipe, ft

To calculate the Reynolds number, all parameters, i.e., ρ, μ, and υ, related to the fluid shall be evaluated at the flowing temperature and pressure, converting from standard condition such as 60^{o}F and 1 atmosphere to the flowing conditions.

Most fluid flowing in the in-plant piping or pipelines are in the turbulent flow regime.

Friction Factor f

Darcy friction factor is more commonly used in the engineering calculations.

For **laminar** flow:

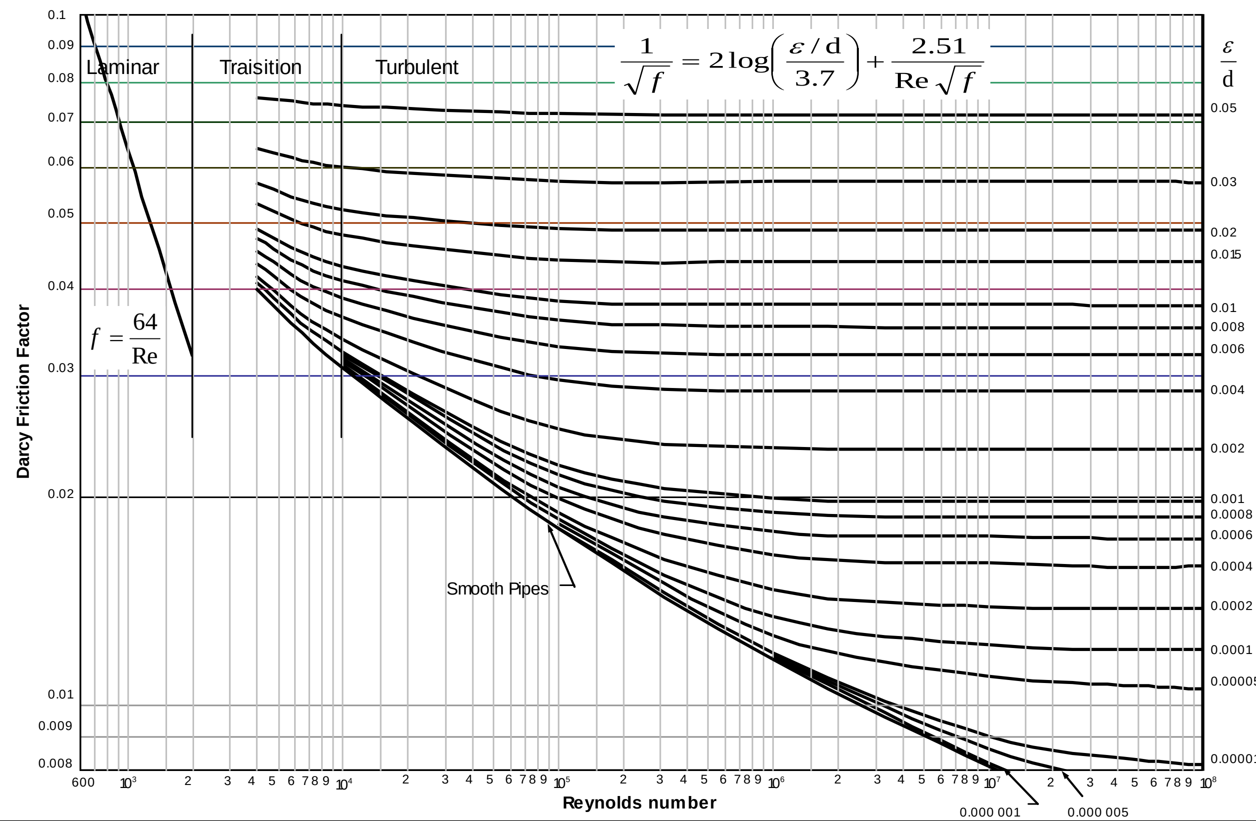
$$f = \frac{64}{\mathrm{Re}}. \qquad (26)$$

For **turbulent** flow, Moody's chart is generally accepted for the friction factor calculation. The Colebrook-White equation is an analytical form for the Moody's friction factor:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\varepsilon/\mathrm{d}}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right] \qquad (27)$$

The Cole Brook-White equation representation of Moody's friction factor is plotted in Fig. 1.

Fig. 1 Darcy Friction Factor (Moody's chart represented by equation)



Absolute Roughness of Pipe ε

For turbulent flow, Moody's friction is correlated with both Reynolds number and the pipe's absolute roughness. Typical values are listed below for new pipes:

Table 1

	Recommended	Absolute Roughness ε
Pipe or Tubing Material	feet	*inches*
Drawn Tubing	0.000 005 [1]	*0.000 06*
Teflon, PTFE; PFA-PTFE	0.000 005	*0.000 06*
High Density Polyethylene, PVDF; PVC; etc	0.000 07	*0.000 84*
Epoxy Coated Steel	0.000 025	*0.000 3*
New API Line Pipe	0.000 0582	*0.000 7*
Commercial Steel	0.000 15 [1]	*0.001 8*
Stainless Steel	0.000 15	*0.001 8*
Asphalt Coated Cast Iron	0.000 4 [1]	*0.004 8*
Galvanized Iron	0.000 5 [1]	*0.006*
Cast Iron	0.000 85 [1]	*0.010 2*
Wood Stave	0.003 to 0.000 6 [1]	*0.036 to 0.0072*
Concrete	0.01 to 0.001 [1]	*0.12 to 0.012*
Riveted Steel	0.03 to 0.003 [1]	*0.36 to 0.036*

PTFE= poly(tetrafluoroethylene)
PFA=poly(fluoroalkoxy)

PVC=poly(vinyl chloride)
PVDF=poly(vinylidene fluoride)

Absolute roughness can increase over time after initial service, resulting in higher frictional pressure drop. For example, it can double in 10 to 15 years for petroleum residues and in 25 to 35 years for light distillates. Whenever possible, a back calculated absolute roughness from actual pipe pressure drop data would be preferred.

Darcy Friction Factor for a commercial pipe at Complete Turbulence(1) :

Table 2

Pipe Size	**1"**	**1.5"**	**2"**	**3"**	**4"**	**6"**	**8" to 10"**	**12" to 16"**	**18" to 24"**
Darcy Friction Factor	0.023	0.021	0.019	0.018	0.017	0.015	0.014	0.013	0.012

Frictional Pressure Drop in Pipe

We will introduce three commonly used methods to calculate the pipe frictional pressure drop for gas.

- **Method 1 – Rational-Formula Isothermal Compressible Flow Approach**

  Treat the gas as an ideal gas (i.e., $: z = 1$) flowing in pipe with changing density along the pipe (which effects the acceleration) to be accounted for by fluid's isothermal expansion via Eq. (1):

$$\left(\frac{f\mathrm{L}_{\mathrm{ft}}}{\mathrm{D}_{\mathrm{ft}}} + \sum_i K_i\right) = \frac{1}{\mathrm{kM}_1^{\,2}}\left[1 - \left(\frac{\mathrm{P}_{2,\ psia}}{\mathrm{P}_{1,\ psia}}\right)^2\right] - \ln\left(\frac{\mathrm{P}_{1,\ psia}}{\mathrm{P}_{2,\ psia}}\right)^2 \quad \text{for inlet Mach number } \mathrm{M}_1 \quad (28)$$

or

$$\left(\frac{f\mathrm{L}}{\mathrm{D}} + \sum_i K_i\right) = \frac{1}{\mathrm{kM}_2^{\,2}}\left[\left(\frac{\mathrm{P}_{1,\ psia}}{\mathrm{P}_{2,\ psia}}\right)^2 - 1\right] - \ln\left(\frac{\mathrm{P}_{1,\ psia}}{\mathrm{P}_{2,\ psia}}\right)^2 \quad \text{for outlet Mach number } \mathrm{M}_2 \quad (28a)$$

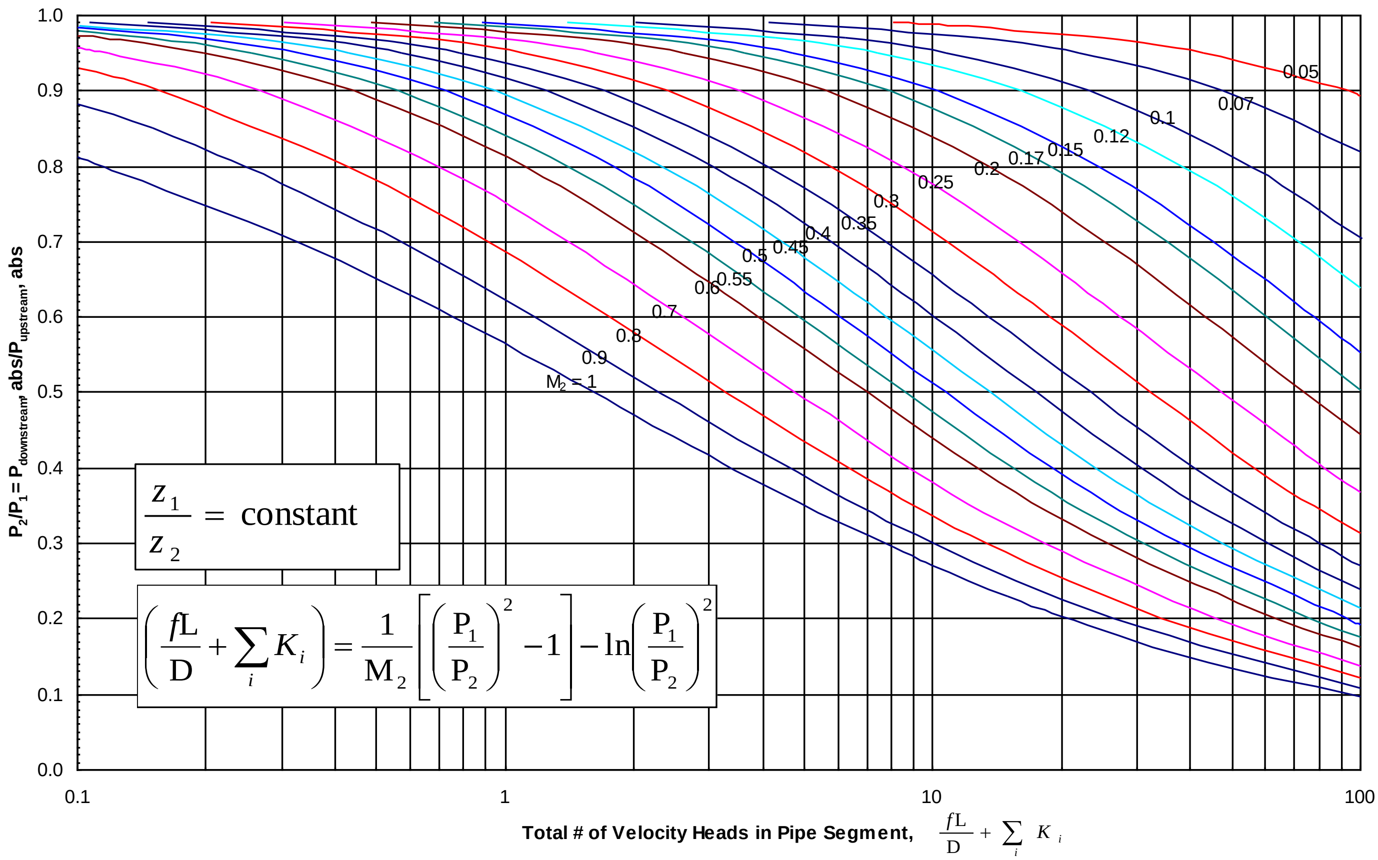
where

$\sum_i K_i$ = Summation of **Resistance Coefficient** or **Number of Velocity Heads** for equipment, valves and fittings that are attached in that pipe segment.

The graphical form of Eq (28a) are illustrared below for a fluid of k=1:

Fig. 2 Isothermal Compressible Flow in Pipe[1,3]

(k = Cp/Cv = 1)



Similar graphs of k= 1.3 and 1.6 can be found, for example, in Ref .3.

Sonic Pressure in Pipe Flow by the Rational Formula

At any given pipe inlet pressure P_1 and piping configuration $\left(\frac{fL}{D} + \sum_i K_i\right)$, the maximum pipe outlet pressure (called Sonic Pressure or the Critical Pressure in pipe, P_{cf}) where the sonic flow occurs can be obtained by specifying the Mach number at the pipe outlet $M_2 = 1$ in Eq (28a):

$$\left(\frac{fL}{D} + \sum_i K_i\right) = \frac{1}{k}\left[\left(\frac{P_{1,\ psia}}{P_{cf,\ psia}}\right)^2 - 1\right] - \ln\left(\frac{P_{1,\ psia}}{P_{cf,\ psia}}\right)^2 \qquad (29a)$$

P_{cf} can be sloved from Eq (29a) by trial-and-error. Alternatively, the sonic pressure figure can also be read off from the $M_2 = 1$ curve in Fig. 2.

Frictional Pressure Drop ΔP_f from Rational Formula

Eq (28) or (28a) can be re-arranged to express the frictional pressure drop $\Delta P_f = P_1 - P_2$ as follows:

$$\Delta P_{f,\ psi} = P_{1,\ psia} - P_{2,\ psia} = \left[\frac{fL_{ft}}{D_{ft}} + \sum_i K_i + \ln\left(\frac{P_{1,\ psia}}{P_{2,\ psia}}\right)^2 \right] \frac{\left(P_{1,\ psia}\right)^2 kM_1^{\ 2}}{2P_{avg,\ psia}} \qquad (30)$$

$$\Delta P_{f,\ psi} = P_{1,\ psia} - P_{2,\ psia} = \left[\frac{fL_{ft}}{D_{ft}} + \sum_i K_i + \ln\left(\frac{P_{1,\ psia}}{P_{2,\ psia}}\right)^2 \right] \frac{\left(P_{2,\ psia}\right)^2 kM_2^{\ 2}}{2P_{avg,\ psia}} \qquad (30a)$$

where $P_{avg,\ psia} = \dfrac{P_{1,psia} + P_{2,psia}}{2}$.

To solve for ΔP_f or P_2 from Eq (30) or (30a), an trial-and-error solution is also needed.

An alternate graphical approach can be used. First, assume a pipe outlet pressure P_2, calculate the outlet Mach number M_2 and calculate the total velocity head of pipe plus valves and fittings $\frac{fL}{D} + \sum_i K_i$. Then, read off $\frac{P_2}{P_1}$ from Fig. 2. Obtain a calculated P_2 (in absolute, psia) by multiplying the inlet pressure P_1 (in absolute, psia) to this ratio. Iterate until the assumed and the calculated P_2 agree. Finally, $\Delta P_{f,\ psi} = P_{1,\ psia} - P_{2,\ psia}$.

Long Gas Pipe Line – A Simplified Case from the Rational-Formula

For a long pipe line where the friction due to line is more significant than that due to the pipe bends or fittings and the line is sized to optimize the line pressure drop, then $\frac{fL}{D} + \sum_i K_i \approx \frac{fL}{D} >> \ell n \left(\frac{P_1}{P_2}\right)^2$ and Eq (28) can be simplified to become:

$$\frac{fL_{ft}}{D_{ft}} = \frac{1}{kM_1^{\ 2}} \left[\frac{\left(P_{1,\ psia}\right)^2 - \left(P_{2,\ psia}\right)^2}{\left(P_{1,\ psia}\right)^2} \right] \qquad (31)$$

Eq (31) can be re-written in terms of mass or volumetric flow rates:

$$\frac{\left(P_{1,psia}\right)^2 - \left(P_{2,psia}\right)^2}{\left(P_{1,\ psia}\right)^2} = \left(\frac{fL_{ft}}{D_{ft}}\right) kM_1^{\ 2}$$

$$= \frac{2}{\left(1891\right)^2}\left(\frac{fL_{ft}}{D_{ft}}\right)\frac{(W_{lb/hr})^2}{\left(P_{1,\ psia}\right)\left(\rho_{1,lb/CF}\right)(d_{inch})^4}$$

$$= \left(6.712x10^{-6}\right)\frac{(f)\left(L_{ft}\right)\left(W_{lb/hr}\right)^2}{\left(P_{1,\ psia}\right)\left(\rho_{lb/CF)}\right)\left(d_{inch}\right)^5} \quad (31a)$$

$$\left(P_{1,psia}\right)^2 - \left(P_{2,psia}\right)^2 = \left(14.52x10^{-9}\right)\frac{(f)\left(L_{ft}\right)(z)\left(T_{o_R}\right)\left(S_g\right)(q_{scfh})^2}{(d_{inch})^5} \quad (31b)$$

$$\left(P_{1,psia}\right)^2 - \left(P_{2,psia}\right)^2 = (25.3)\frac{(f)\left(L_{ft}\right)(z)\left(T_{o_R}\right)\left(S_g\right)(Q_{MMscfd})^2}{(d_{inch})^5} \quad (31c)$$

Eq (31a), (31b) and (31c) are a little akward to use, but can be further simplified to more useful working forms in terms of average line pressure as defined previously:

$$\Delta P_{f,\ psi} = \left(\frac{fL_{ft}}{D_{ft}}\right)\frac{\left(P_{1,\ psia}\right)^2 kM_1^{\ 2}}{2P_{avg,\ psia}}$$

$$= \frac{1}{\left(1891\right)^2}\left(\frac{fL_{ft}}{D_{ft}}\right)\frac{P_{1,\ psia}}{P_{avg,\ psia}}\frac{(W_{lb/hr})^2}{(\rho_{1,lb/CF})(d_{inch})^4}$$

$$= \left(3.356x10^{-6}\right)\frac{P_{1,\ psia}}{P_{avg,\ psia}}\frac{(f)\left(L_{ft}\right)(W_{lb/hr})^2}{(\rho_{1,lb/CF})(d_{inch})^5} \quad (31d)$$

$$\Delta P_{f\ psi} = \left(7.26x10^{-9}\right)\frac{(f)\left(L_{ft}\right)(z)\left(T_{o_R}\right)\left(S_g\right)(q_{scfh})^2}{(P_{avg,\ psia})(d_{inch})^5} \quad (31e)$$

$$\Delta P_{f\ psi} = (12.65)\frac{(f)\left(L_{ft}\right)(z)\left(T_{o_R}\right)\left(S_g\right)(Q_{MMscfd})^2}{(P_{avg,\ psia})(d_{inch})^5} \quad (31f)$$

Eq (31) and its variations such as (31a), (31b), (31c), (31d), (31e) and (31f) are called **rational formula for gas pipe lines**

- **Method 2 – Darcy's Equation Isothermal Incompressible Flow Approach**
  The Darcy's isothermal incompressible flow equation for liquid is used as approximation to the gas application:

$$\Delta P_{f_{psi}} = \left(f \frac{L_{ft}}{D_{ft}} + \sum_i K_i \right) \frac{(\rho_{1,\ lb/CF})(\upsilon_{1,\ ft/\sec})^2}{(2)(g_c)(144)} \qquad (32)$$

where the subscript "1" refers to the pipe inlet conditions.

While the kinetic energy losses caused by acceleration of fluid in a pipe is generally negligible in liquid flows, they can be important for gas flows whenever the pressure drop (or gas density) undergoes a significant change. Therefore, Darcy's overall frictional isothermal pressure drop equation, as typically used for fluids flowing at constant density and velocity, does not consider the fluid expansion but can be used with understanding of its applicable limit and with caution. Refer to the next section on comparison of accuracy with the rational flow equations.

We shall also note that Darcy's incompressible flow equation, Eq (32), does not have a mechanism to deal with the critical flow.

One variation of Eq (32) is to rewrite it in terms of mass flow rate. For example, Eq (32) can be rearranged to take the following forms in terms of mass flow rate:

$$\Delta P_{f\ psi} = \frac{1}{(1891)^2} \left(\frac{f L_{ft}}{D_{ft}} + \sum_i K_i \right) \frac{(W_{lb/hr})^2}{(d_{\ inch})^4 (\rho_{1,\ lb/CF})} \qquad (32a)$$

or

$$W_{lb/hr} = 1891 (d_{inch})^2 \sqrt{\frac{\Delta P_{f\ psi} \rho_{1,\ lb/CF}}{\left(\frac{f L_{ft}}{D_{ft}} + \sum_i K_i \right)}} \qquad (32b)$$

Long Gas Pipe Line – A Simplified Case from Darcy's Equation
Furthermore, Eq (32) can also be simplified somewhat with the assumption that the number of velocity head for lines is far greater than that for valves and fitting on that line, i.e., $f \frac{L_{ft}}{D_{ft}}. >> \sum_i K_i$, for application to long pipe lines:

$$\Delta P_{f_{psi}} = \left(f \frac{L_{ft}}{D_{ft}} \right) \frac{(\rho_{1,\ lb/CF})(\upsilon_{1,\ ft/\sec})^2}{(2)(g_c)(144)} = \frac{1}{(1891)^2} \left(\frac{f L_{ft}}{D_{ft}} \right) \frac{(W_{lb/hr})^2}{(\rho_{1,\ lb/CF})(d_{inch})^4} \qquad (33)$$

or

$$\Delta P_{f\ psi} = \left(7.26x10^{-9}\right)\frac{(f)\left(L_{ft}\right)(z)\left(T_{1,\ ^oR}\right)\left(S_g\right)(q_{scfh})^2}{(P_{1,\ psia})(d_{inch})^5} \tag{33a}$$

$$\Delta P_{f\ psi} = (12.65)\frac{(f)\left(L_{ft}\right)(z)\left(T_{1,\ ^oR}\right)\left(S_g\right)(Q_{MMscfd})^2}{(P_{1,\ psia})(d_{inch})^5} \tag{33b}$$

in terms of inlet flowing parameters and standard volumetric flow rate.

Notice that all flowing parameters applying to Darcy's equation are evaluated at the pipe entrance conditions. Potential applicable flow situation using Darcy's equation is for short piping runs without significant heat transfer to or from pipe.

- **Method 3 - Darcy's Isothermal Equation with Net Expansion Factor**
  To account for the effect of gas expansion and acceleration when using Darcy's frictional pressure drop equation, a correction factor can be incorporated. The correction is achieved by incorporating an additional parameter, called Net Expansion Factor (Y), into Eq (32) using the flowing conditions at pipe inlet:

$$\Delta P_{f\ \ psi} \equiv \left(\frac{fL_{ft}}{D_{ft}} + \sum_i K_i\right)\left(\frac{1}{Y^2}\right)\frac{(\rho_{1,\ lb/CF})(\upsilon_{1,\ fps})^2}{(2)(g_c)(144)} \tag{34}$$

Similarly, the dynamic head loss can also be expressed in terms of flow rates as follows:

$$\Delta P_{f\ \ psi} = \frac{1}{(1891)^2}\frac{1}{Y^2}\left(\frac{fL_{ft}}{D_{\ ft}} + \sum_i K_i\right)\frac{(W_{lb/hr})^2}{(d_{inch})^4(\rho_{1,\ lb/CF})} \tag{34a}$$

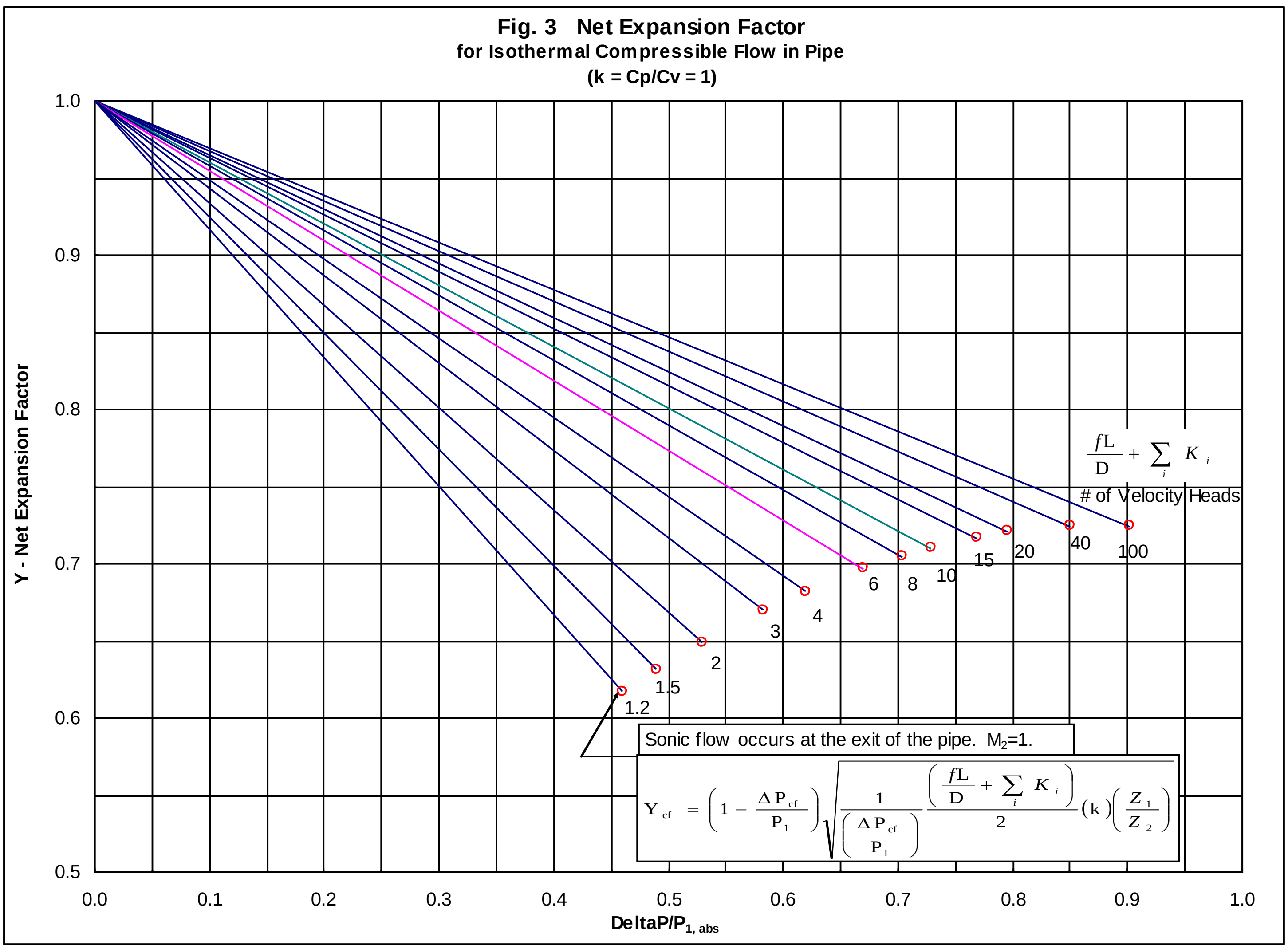
or

$$W_{lb/hr} = (1891)(Y)(d_{\ inch})^2\sqrt{\frac{\Delta P_{f\ \ psi}\rho_{1,\ lb/CF}}{\left(\frac{fL_{ft}}{D_{1\ \ ft}} + \sum_i K_i\right)}} \tag{34b}$$

The Y_{cf} is obtained by solving Eq. (34) at the sonic conditions ($\Delta P = \Delta P_{cf}$, $Y = Y_{cf}$) with the help of Eq (1) the density equation, and Eq (4) the mass balance equation:and is given by Eq. (35):

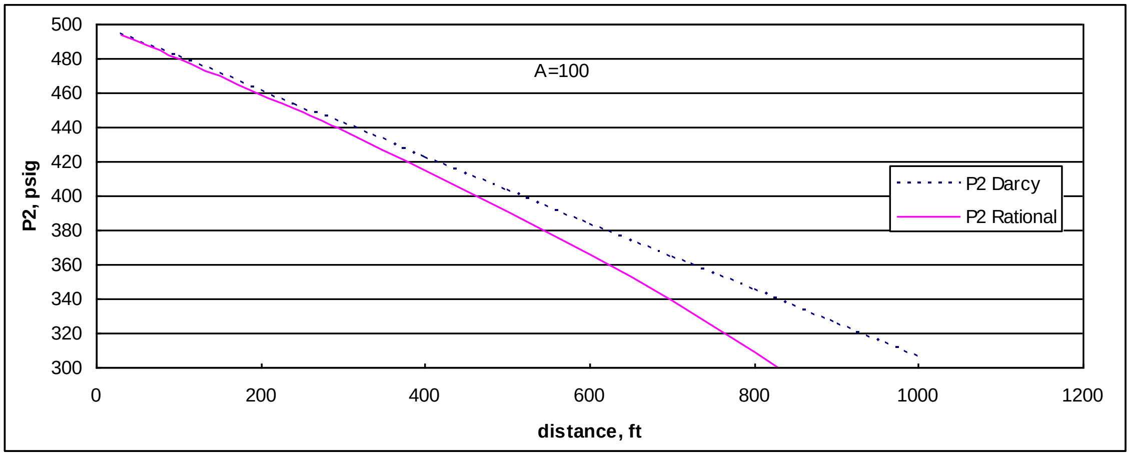
$$Y_{cf} = \left(1 - \frac{\Delta P_{cf}}{P_1}\right)\sqrt{\frac{1}{\left(\frac{\Delta P_{cf}}{P_1}\right)} \frac{\left(\frac{fL}{D} + \sum_i K_i\right)}{2} (k)\left(\frac{z_1}{z_2}\right)} \quad (35)$$

A plot of Y vs. $\left(\frac{\Delta P}{P_1}\right)$ is illustrated as an example in Fig. 3 for a fluid of k=1. The Net Expansion Factor Y can be estimated by linear interpolation between $Y = 1$ and that at the critical condition $Y = Y_{cf}$. The pressure at which the critical flow occurs can be estimated by Eq (29a) or from the $M_2 = 1$ curve in Fig. 2.



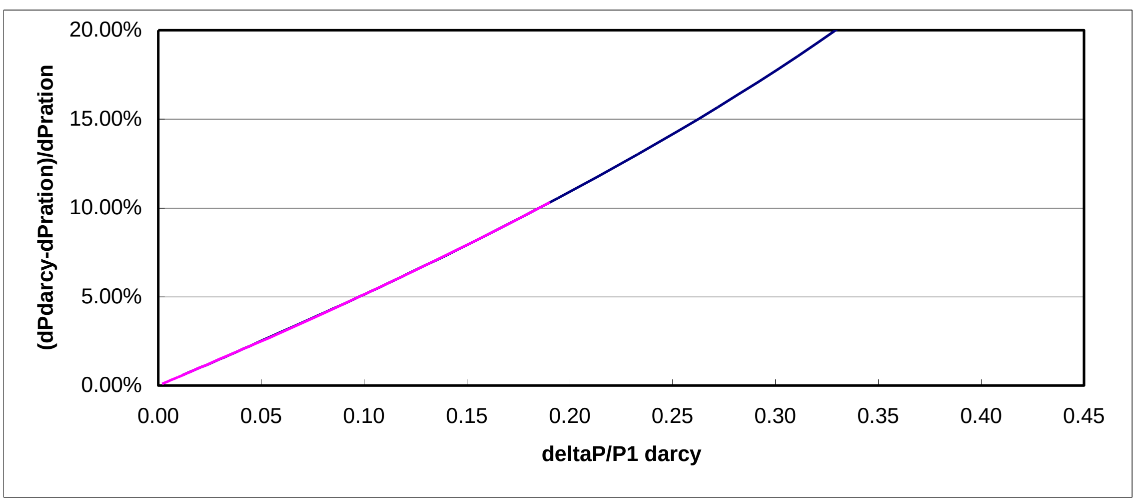
Comparison between the Rational and Darcy's methods

Let's now examine if there is any difference in the result of ΔP_f calculation by the Darcy-based incompressible flow approach Eq (33) and the isothermal compressible flow rational formula Eq (31). Both of these two equations are plotted below for an illustration case with the lumped parameter A of 100, where $A = \left(7.26\text{x}10^{-9}\right)\frac{(f)\left(L_{ft}\right)(z)\left(T_{o_R}\right)\left(S_g\right)\left(q_{scfh}\right)^2}{\left(d_{inch}\right)^5}$



As expected, the Darcy's formula calculates a higher outlet pressure (i.e., lower line pressure drop) because the flow density was not allowed to change along the pipe and, hence, no pressure decrease due to gas expansion was accounted.

The percent deviation of calculated pressure drop of Darcy's incompressible formula from the rational formula is illustrated below:



As one can see that the deviations become large when the calculated pressure drop by Darcy's equation is high relative to the pipe entrance pressure. To allow the deviation of the Darcy's to be below reasonable range, it is recommended that the simpler Darcy's incompressible equation be used when the following criterion is met:

Total segment frictional pressure drop $\leq$10% of the segment entrance pressure.

Empirical Formula

A few commonly used empirical formula in the gas industry are illustrated in this section. These formula can be oberved as variations from a simplified isothermal compressible flow with uniqic friction factor expression. The use of isothermal compressible pipe flow formulation is a reasonable approach because the flowing temperature of the relatively slower gas flowing velocity of a cross-country long pipeline tends to reach the ambient temperature.

- **Panhandle A**

Substituting the Dracy's friction factor in the Rational formula for long gas pipe line, Eq. (31c), for example, with

$$f = 0.04835\left(\frac{d_{inch}}{q_{scfh}S_g}\right)^{0.1461} E^{-1.8539}, \quad (36)$$

one obtains

$$Q_{SCFD} = (435.87E)\left(\frac{T_{b,520^oR}}{P_{b,14.69psia}}\right)(d_{inch})^{2.6182}\left(\frac{P_{1,\,psia}{}^2 - P_{2,\,psia}{}^2}{S_g{}^{0.8539}L_{mile}zT_{^oR}}\right)^{0.5394} \quad (37)$$

or, substituting Dracy's friction factor in Eq (31b), and subsequently letting S_g =0.6 & $T_{^oR}$ = 60 + 460 = 520 flowing temperature for a special condition:

$$q_{scfh} = (36.8E)(d_{inch})^{2.6182}\left(\frac{P_{1,psia}{}^2 - P_{2,psia}{}^2}{zL_{mile}}\right)^{0.5394} \quad (38)$$

for the Panhandle A formula for cross-country pipe lines, where,

E = flow efficiency
E = 1.0 for brand new pipe without any bends, elbows, valves, and chages of pipe diameters or elevations
E = 0.95 for very good operating conditions
E = 0.92 for average operating conditions
E = 0.85 for very unusally unfavorable operating conditions.

- **Weymouth**

Substituting Eq. (31c) and (31b), respectively, with Dracy's friction factor

$$f = \frac{0.032}{(d_{inch})^{1/3}} \quad (39)$$

one obtains

$$Q_{SCFD} = (443.45)\left(\frac{T_{b,520^{o}R}}{P_{b,14.69psia}}\right)(d_{inch})^{2.667}\left(\frac{P_{1,\ psia}{}^{2} - P_{2,\ psia}{}^{2}}{S_{g}L_{mile}zT_{^{o}R}}\right)^{0.5} \qquad (40)$$

or

$$q_{scfh} = (28)(d_{inch})^{2.667}\sqrt{\frac{P_{1,psia}{}^{2} - P_{2,psia}{}^{2}}{zL_{mile}S_{g}}\left(\frac{520}{T_{^{o}R}}\right)}, \qquad (41)$$

which are the Weymouth formula for cross-country pipe lines.

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